RISK AND RETURN

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Reference Material: Chapter 12 and 13 of Textbook

TOPICS

1. First Lesson: Average Returns
2. Second Lesson: Variability of Returns
3. Arithmetic vs Geometric Returns
4. Capital Market Efficiency
5. Expected Returns and Variances
6. Portfolios
7. Diversification
    - Total, systematic, unsystematic risk
    - Beta
    - Reward-to-risk Ratio
    - Security Market Line and CAPM

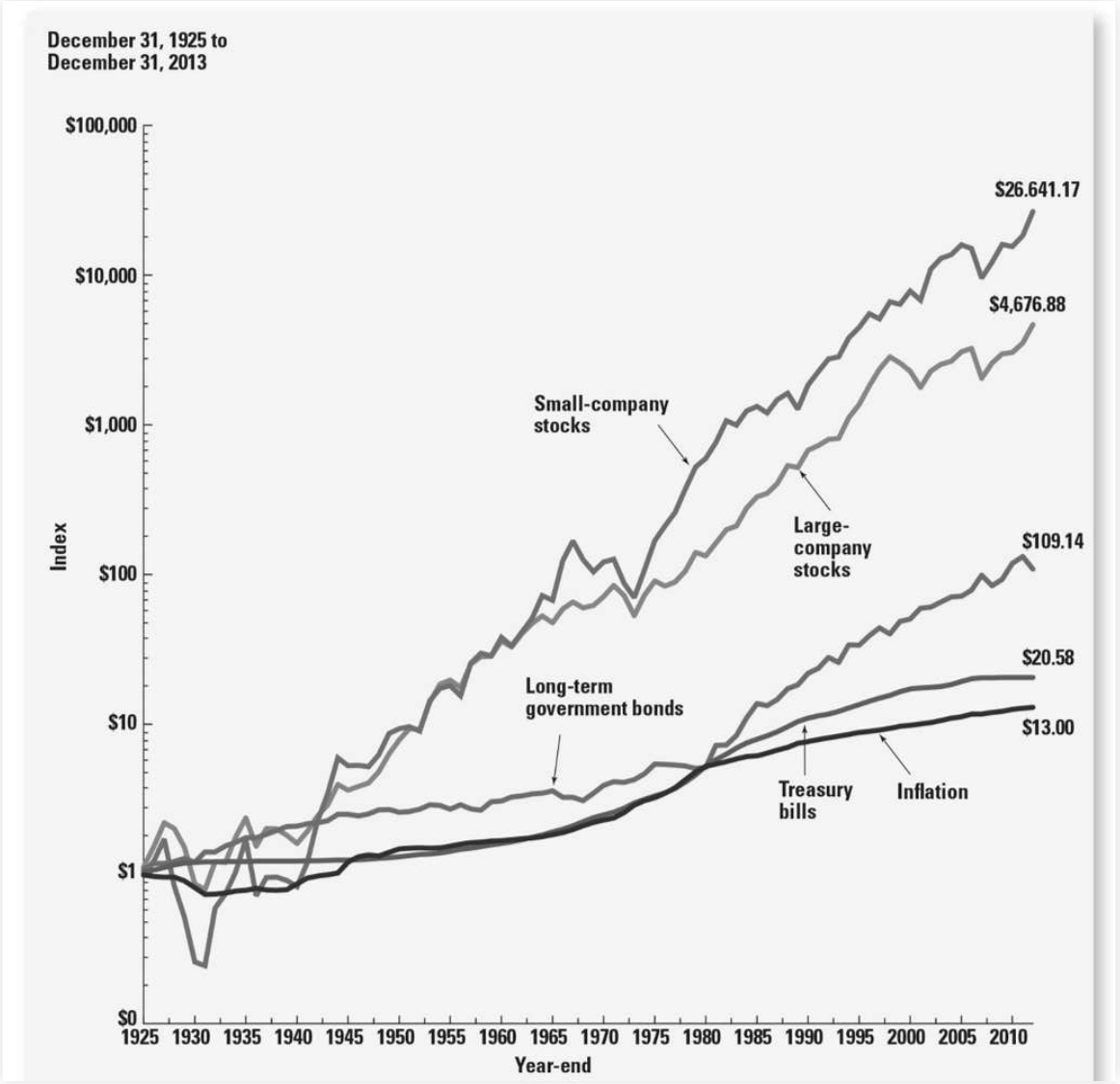
OVERVIEW: RISK, RETURN AND FINANCIAL MARKETS

- Lessons from capital market history
  - There is a reward for bearing risk
  - The greater the potential reward, the greater the risk
  - This is called the risk-return trade-off

How should we measure risk and return?

FIRST LESSON: AVERAGE RETURN

HISTORICAL RECORD



RANKING RETURNS

1. Small cap stocks
2. Large cap stocks
3. Long-term government bonds
4. Treasury Bills
5. Inflation

Why wouldn't you just buy small cap stocks?

RISK!!!!

CALCULATING RETURNS

1. Total Dollar Return

    - $Return = Dividends + Capital Gains

2. Total Percent Return

    - %Return = $\frac{\$Return}{\$Invested}$

EXAMPLE: RETURNS

You just invested in "You call that a Donut! Inc" for $25, after one-year the price is $35. Each share paid out a $2 dividend. What was your total return?

	Dollar Return	**Percent Return**
Dividend	2	$\frac{2}{25} = 8\%$
Capital Gains	35-25=10	$\frac{35-25}{25} = 40\%$
Total Return	**2+10=12**	$\mathbf{\frac{10+2}{25} = 48\%}$

PERCENT RETURNS: FORMULAS

Dividend Yield

$$DY = \frac{D_{t+1}}{P_t}$$

Capital Gains Yield

$$CGY = \frac{P_{t-1} - P_t}{P_t}$$

$$\%Return = \frac{D_{t+1} + P_{t+1} - P_t}{P_t}$$

HISTORICAL AVERAGE RETURNS

$$HistoricalAverageReturn = \frac{\sum_{i=1}^{T} Return_i}{T}$$

Large cap stocks average return from 1926 to 2010: 12.1%

Your best guess about the size of the return for a year selected at random is 12.1%.

HISTORICAL AVERAGE RETURNS: 1926-2010

Investment	Average Return
Large-company stocks	12.1%
Small-company stocks	16.9
Long-term corporate bonds	6.3
Long-term government bonds	5.9
U.S. Treasury bills	3.5
Inflation	3.0

PRACTICE: AVERAGE

Returns: -6, 8, 12, -15, 6

Average = 1

Returns: -1, 2, -1, 1, 4

Average = 1

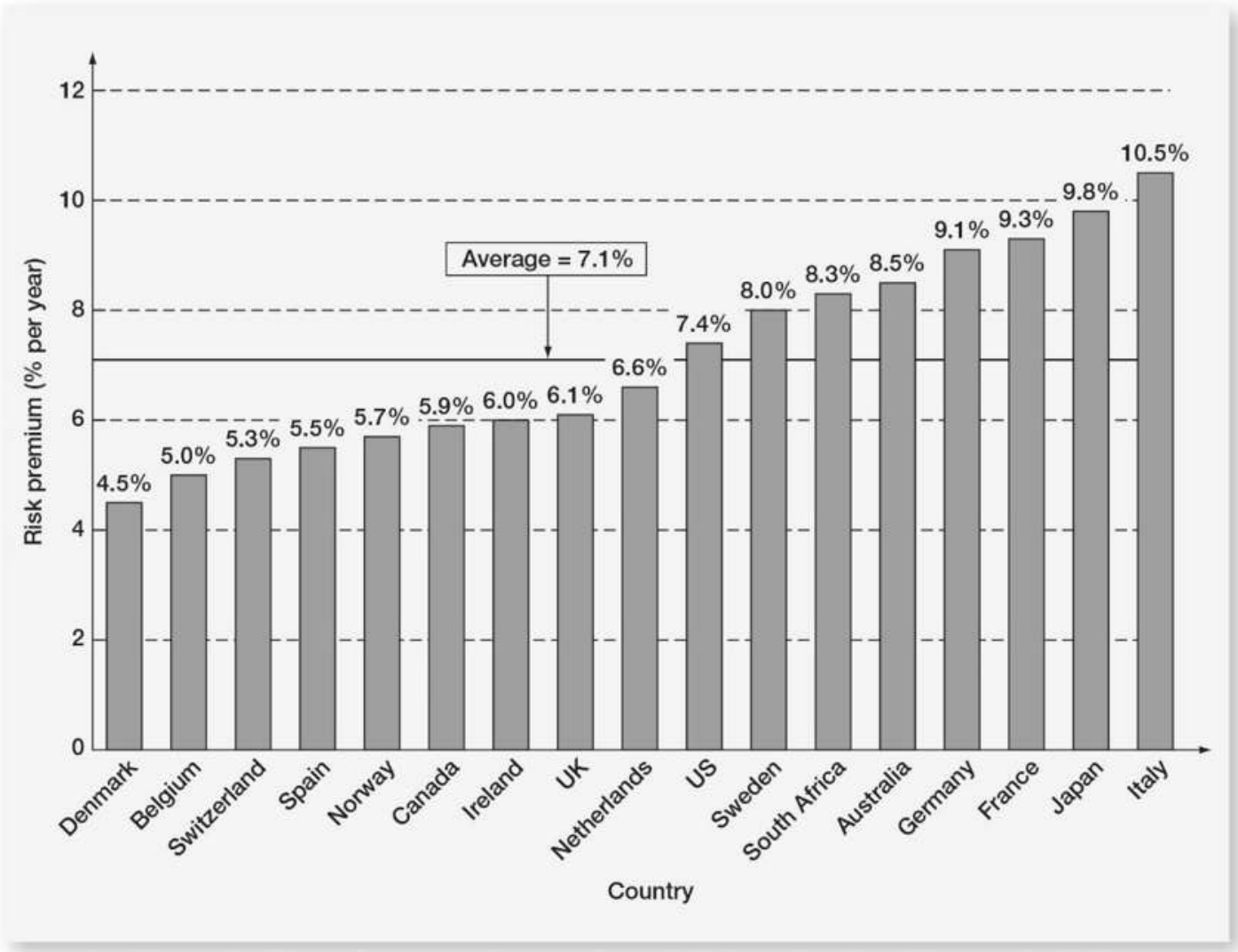
RISK PREMIUM

The excess return required from an investment in a risky asset over that required from a risk free investment.

Investment	Average Return	Risk Premium
Large-company stocks	12.1%	8.6%
Small-company stocks	16.9	13.4
Long-term corporate bonds	6.3	2.8
Long-term government bonds	5.9	2.4
U.S. Treasury bills	3.5	.0

U.S. Treasury bill is considered risk-free return

FIGURE 12.13 Stock Market Risk Premiums for 17 Countries: 1900–2005



SOURCE: Based on information in Elroy Dimson, Paul Marsh, and Michael Staunton, "The Worldwide Equity Premium: A Smaller Puzzle," in *Handbook of the Equity Risk Premium*, Rajnish Mehra, ed. (Elsevier: 2007).

FIRST LESSON TAKEAWAYS

Risky assets, on average, earn a risk premium

Large company stocks have a historical average risk premium of 8.6%

What determines size of risk premium?

SECOND LESSON: RETURN VARIABILITY

MEASURING RETURN VARIABILITY

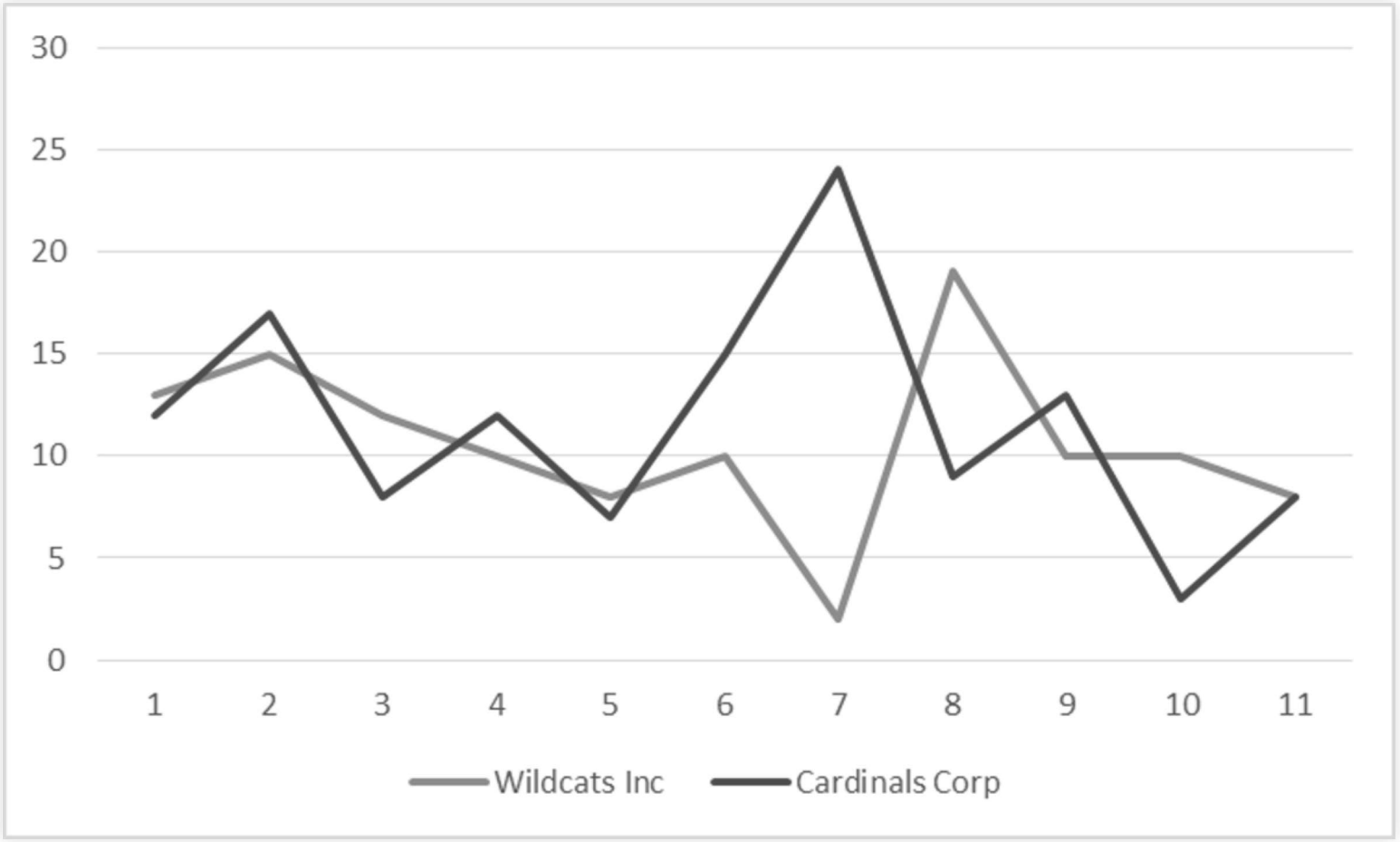
- Variance or σ^2
  - Common measure of return dispersion
- Standard deviation or σ
  - Sometimes called volatility
  - Same "units" as the average

EXAMPLE

Two companies have the following returns:
Wildcat Inc: 13,15,12,10,8,10,2,19,10,10,8
Cardinals Corp: 12,17,8,12,7,15,24,9,13,3,8

	Wildcats Inc.	**Cardinals Corp.**
Average	10.6	11.6
Standard Deviation	4.3	5.7

GRAPHICAL REPRESENTATION



STEPH VS LEBRON (POINTS IN 2016 PLAYOFFS)

Steph Curry (9 games leading into finals):40,29,26,28,24,19,31,31,36.
LeBron James (9 games leading into finals):27,24,21,24,23,24,29,23,33.

	Steph	**LeBron**
Average	29.33	25.33
Standard Deviation	6.25	3.71

RETURN VARIABILITY

- Return Variance:

$$VAR(R) = \sigma^2 = \frac{\sum_{i=1}^{T}(R_i - \bar{R})^2}{T-1}$$

- Standard deviation:

$$STD(R) = \sigma = \sqrt{VAR(R)}$$

PRACTICE: STANDARD DEVIATION

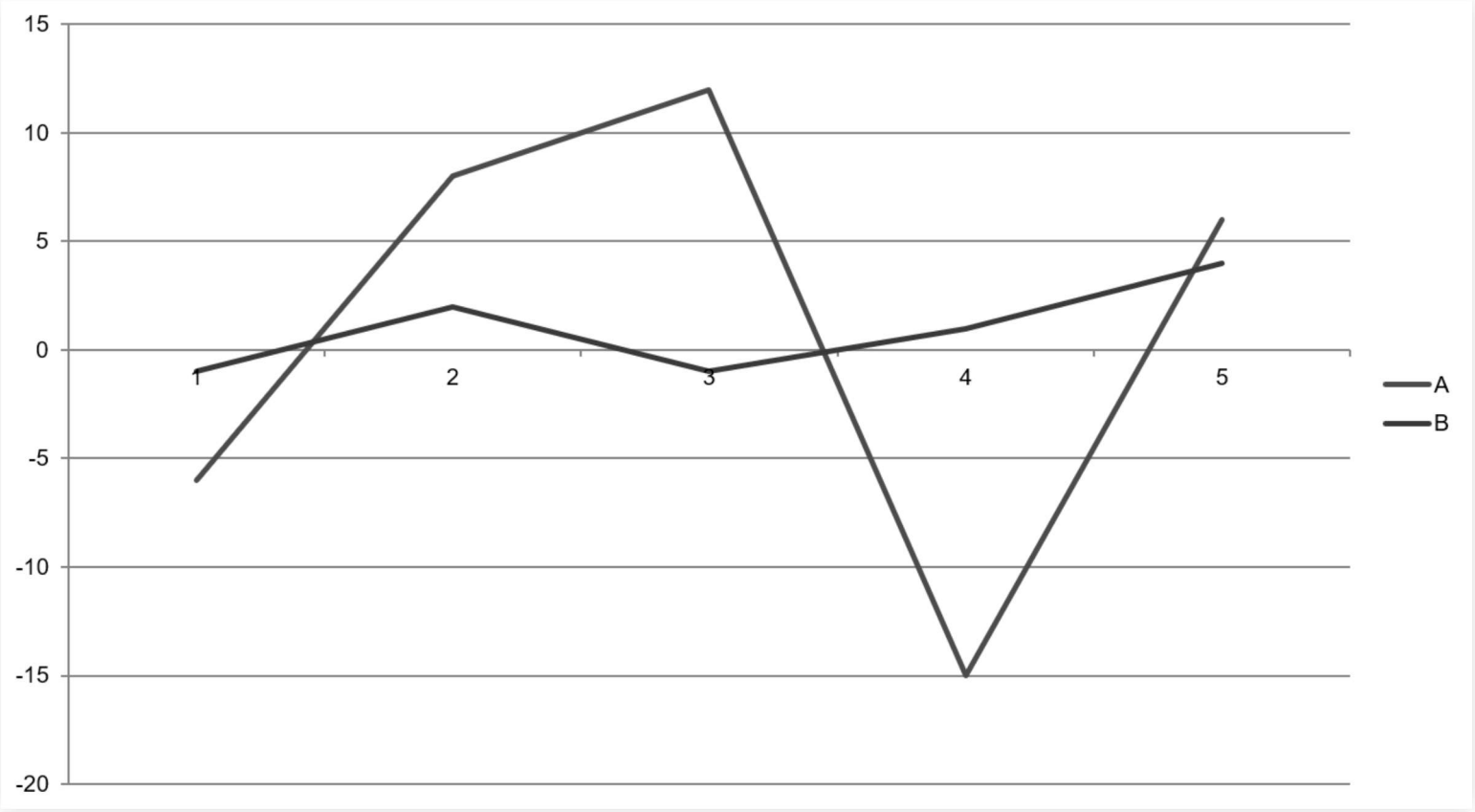
Returns(A): -6, 8, 12, -15, 6

Average = 1
Standard deviation = 11.18

Returns(B): -1, 2, -1, 1, 4

Average = 1
Standard deviation = 2.12

GRAPHING RETURNS

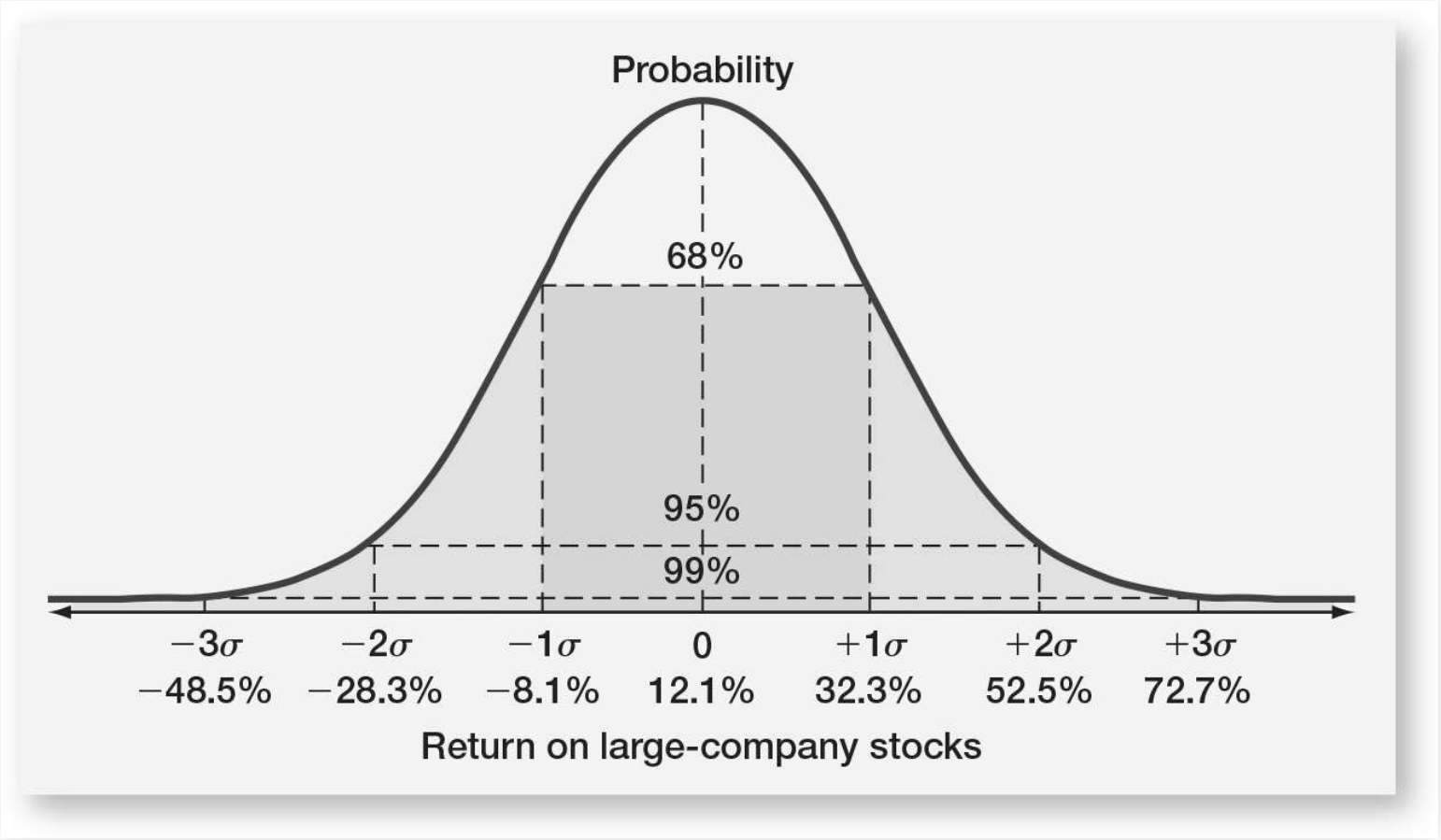


EXAMPLE

Year	Return (%)	Average Return (%)	Difference	Squared Difference
1926	11.14	11.48	-.034	0.0012
1927	37.13	11.48	25.65	657.82
1928	43.31	11.48	31.83	1013.02
1929	-8.91	11.48	-20.39	415.83
1930	-25.26	11.48	-36.74	1349.97
			Variance	859.19
			Standard Deviation	29.31

NORMAL DISTRIBUTION

A symmetric bell-shaped frequency distribution that is completely defined by its mean and standard deviation.



ARITHMETIC VS. GEOMETRIC MEAN

THINK ABOUT RETURNS...

If you invest in a hedge fund that loses 20% the first year, but makes 20% the second year, are you back to even?

- **NO!!!!!**
- Start with $100
- After year 1: you have $80
- After year 2, you have $96

ANOTHER EXAMPLE

Suppose you invest $100 and it falls 50% in year one but gain 100% in year 2.

- Year 0:100
- Year 1:100*(1-0.50)=50
- Year 2:50*(1+1)=100

ARITHMETIC VS. GEOMETRIC MEAN

- Arithmetic average:
  - Return earned in an average period over multiple periods
  - Answers the question: "What was your return in an average year over a particular period?"
- Geometric average
  - Average compound return per period over multiple periods
  - Answers the question: "What was your average compound return per year over a particular period?"

Geometric average < Arithmetic average unless all the returns are equal

GEOMETRIC AVERAGE: FORMULA

$$GAR = [(1 + R_1) * (1 + R_2) * \ldots * (1 + R_T)]^{\frac{1}{T}} - 1$$

Where:

R_i = return in each period

T = number of periods

GEOMETRIC AVERAGE: FORMULA

$$GAR = [\prod_{i=1}^{T}(1 + R_i)]^{\frac{1}{T}} - 1$$

Where:

$\prod$ = Symbol for product (multiply)

R_i = return in each period

T = number of periods in sample

REVISIT EXAMPLES

If you invest in a hedge fund that loses 20% the first year, but makes 20% the second year.
Average Return: 0%
Geometric return: -2.02%

Suppose you invest $100 and it falls 50% in year one but gain 100% in year 2.
Average Return: 25%
Geometric Return: 0%

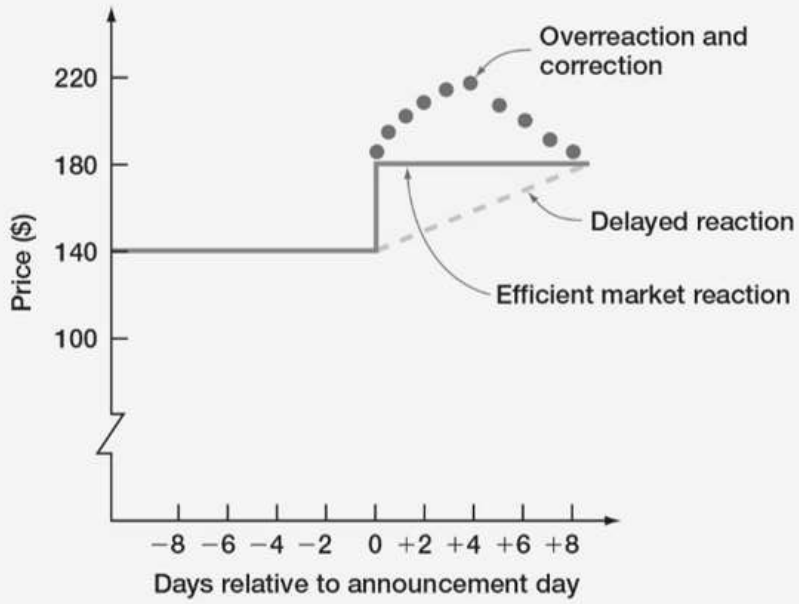
EXAMPLE

Year	Return (%)	(1+R)	Compounded
1926	11.14	1.114	1.114
1927	37.13	1.3713	1.5241
1928	43.31	1.4331	2.1841
1929	-8.91	0.9109	1.9895
1930	-25.26	0.7474	1.4870
		$(1.4870)^{\frac{1}{5}}$	1.0826
		Geometric return	8.26%

CAPITAL MARKET EFFICIENCY

CAPITAL MARKET EFFICIENCY

A market in which security prices reflect available information



Efficient market reaction: The price instantaneously adjusts to and fully reflects new information; there is no tendency for subsequent increases and decreases to occur.
Delayed reaction: The price partially adjusts to the new information; eight days elapse before the price completely reflects the new information.
Overreaction: The price overadjusts to the new information; it overshoots the new price and subsequently corrects.

If true, cannot earn abnormal or excess returns.

Efficient Market Hypothesis Video Investopedia

EFFICIENT MARKET HYPOTHESIS

The hypothesis that actual capital markets are efficient.

- Idea is competition among investors drives information into prices and thus the market becomes more and more efficient.
- Stocks are all priced correctly

FINANCE VERSION OF "DAD JOKE"

A student and a finance professor are walking down the hall when they both see a $20 bill on the ground. The student bends down to pick it up.

The professor shakes their head slowly with a look of disappointment. And says...

"Don't bother, If it were really there, someone else would have picked it up already"

FORMS OF MARKET EFFICIENCY

1. Strong form: all information of every kind is reflected in the stock prices. Including public and private.
2. Semi-strong form: all public information is reflected in stock prices.
3. Weak form: Prices reflect all past trading information such as prices and volume.

SUMMARY

- No simple way to "beat" the market
- Identifying mispriced stocks is very difficult (borderline impossible)
- Prices do respond rapidly to information
- Very difficult to predict future stock prices

EXPECTED RETURNS AND VARIANCES

WEIGHTED AVERAGE REMINDER

Your grade is weighted 30% for the midterm 50% for the final. Homework is worth 10% and quizzes another 10%. You did perfect on the homework and quizzes. The midterm you received a 81 and the final was an 92. What is your final grade?

Answer: 90.3

EXPECTED RETURNS

- Expected returns are based on the probabilities of possible outcomes
- In this context, "expected" means average if the process is repeated many times
- The "expected" return does not even have to be a possible return

$$E(R) = \sum_{i=1}^{N} p_i R_i$$

EXAMPLE: E(R)

Suppose you have predicted the following returns for stocks C and T in three possible states of the economy. What are the expected returns?

State	Probability	C	T
Boom	0.3	0.15	0.25
Normal	0.5	0.1	0.2
Recession	???	0.02	0.01
	Expected Return	9.9%	17.7%

What is the risk premium if the US treasury bill rate is 4.15%?

C=5.75% T=13.55%

VARIANCE AND STANDARD DEVIATION

- Variance and standard deviation measure the volatility of returns
- Using unequal probabilities for the entire range of possibilities
- Weighted average of squared deviations

$$\sigma^2 = \sum_{i=1}^{n} p_i (R_i - E(R))^2$$

EXAMPLE

State	P_i	C	T	$p_i(R_i - E(R))2$	$p_i(R_i - E(R))2$
Boom	0.3	0.15	0.25	0.3(0.15 − 0.099)2	0.3(0.25 − 0.177)2
Normal	0.5	0.1	0.2	0.5(0.1 − 0.099)2	0.5(0.2 − 0.177)2
Recession	0.2	0.02	0.01	0.2(0.02 − 0.099)2	0.2(0.01 − 0.177)2
			σ2	0.002029	0.007441
			σ	4.50%	8.63%

PORTFOLIOS

WHAT IS A PORTFOLIO?

- A portfolio is a collection of assets
- An asset's risk and return are important in how they affect the risk and return of the portfolio
- The risk-return trade-off for a portfolio is measured by the portfolio expected return and standard deviation, just as with individual assets

PORTFOLIO WEIGHTS

Suppose you have $15,000 to invest and you have purchased securities in the following amounts. What are your portfolio weights in each security?

Portfolio	Weights
$2000 of DIS	2/15=13.33%
$3000 of KO	3/15=20%
$4000 of AAPL	4/15=26.7%
$6000 of PG	6/15=40%

PORTFOLIO EXPECTED RETURN

The expected return of a portfolio is the weighted average of the expected returns of the respective assets in the portfolio

$$E(R_p) = \sum_{j=1}^{m} w_j E(R_j)$$

- You can also find the expected return by finding the portfolio return in each possible state and computing the expected value as we did with individual securities

EXAMPLE

Stock	Weight	Return	$w_j E(R_j)$
DIS	.1333	19.69%	2.62%
KO	.20	5.25%	1.05%
AAPL	.267	16.65%	4.45%
PG	.40	18.24%	7.30%
		$E(R_p)$	**15.41%**

PORTFOLIO VARIANCE

1. Compute the portfolio return for each state.
2. Compute the expected portfolio return using the same formula as for an individual asset.
3. Compute the portfolio variance and standard deviation using the same formulas as for an individual asset.

EXAMPLE

State	P_i	A (50%)	B (50%)	$E(R_p)$	$p_i(E(R_j) - E(R_p))2$
Boom	.4	30%	-5%	12.5%	.4(12.5 − 9.5)2 = 3.6
Bust	.6	-10%	25%	7.5%	.6(7.5 − 9.5)2 = 2.4
	$E(R_i)$	6%	13%	$E(R_p)$9.5%	σ^2_p=6
	σ^2_I	384	216		σ_p=2.45%
	σ_i	19.6%	14.7%		

Note: You CANNOT use stock level σ^2 and σ to calculate portfolio.

RISK, RETURN, AND DIVERSIFICATION

SYSTEMATIC RISK

- Risk factors that affect a large number of assets
- Also known as non-diversifiable risk or market risk
- Includes such things as changes in GDP, inflation, interest rates, etc.

UNSYSTEMATIC RISK

- Risk factors that affect a limited number of assets
- Also known as unique risk and asset-specific risk
- Includes such things as labor strikes, part shortages, etc.

RETURNS

$$TotalReturn = ExpectedReturn + UnexpectedReturn$$

$$UnexpectedReturn = SystematicPortion + UnsystematicPortion$$

$$TotalReturn = ExpectedReturn + SystematicPortion + UnsystematicPortion$$

DIVERSIFICATION

Portfolio diversification is the investment in several different asset classes or sectors

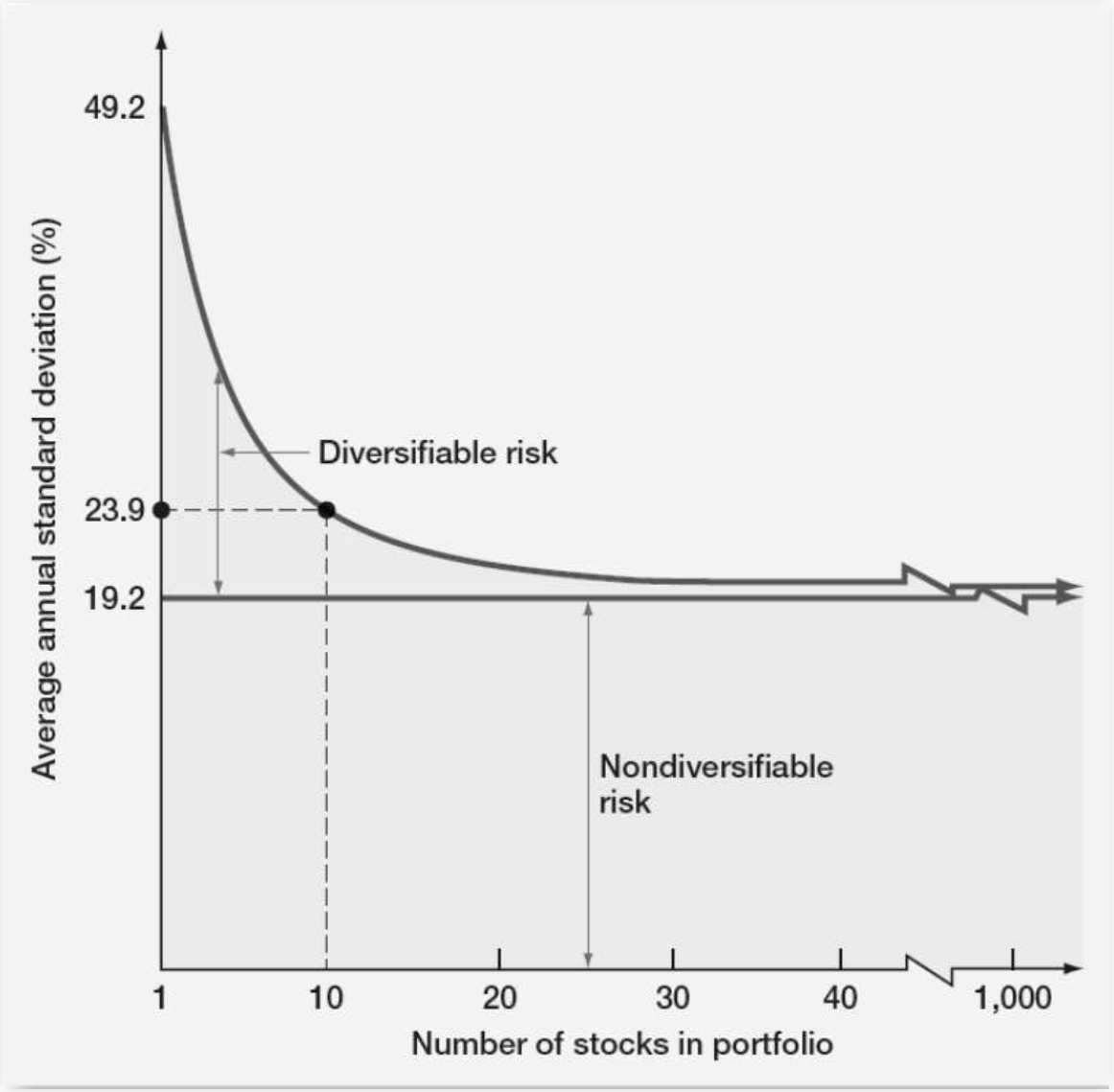
- Diversification is not just holding a lot of assets
- For example, if you own 50 Internet stocks, you are not diversified
- However, if you own 50 stocks that span 20 different industries, then you are diversified

What's Diversification? | Fidelity

THE PRINCIPLE OF DIVERSIFICATION

- Diversification can substantially reduce the variability of returns without an equivalent reduction in expected returns
- This reduction in risk arises because worse than expected returns from one asset are offset by better than expected returns from another
- However, there is a minimum level of risk that cannot be diversified away and that is the systematic portion

DIVERSIFIABLE VS NON-DIVERSIFIABLE RISK



DIVERSIFIABLE RISK

- The risk that can be eliminated by combining assets into a portfolio
- Often considered the same as unsystematic, unique or asset-specific risk
- If we hold only one asset, or assets in the same industry, then we are exposing ourselves to risk that we could diversify away

TOTAL RISK

Total risk = systematic risk + unsystematic risk

- The standard deviation of returns is a measure of total risk
- For well-diversified portfolios, unsystematic risk is very small
- Consequently, the total risk for a diversified portfolio is essentially equivalent to the systematic risk

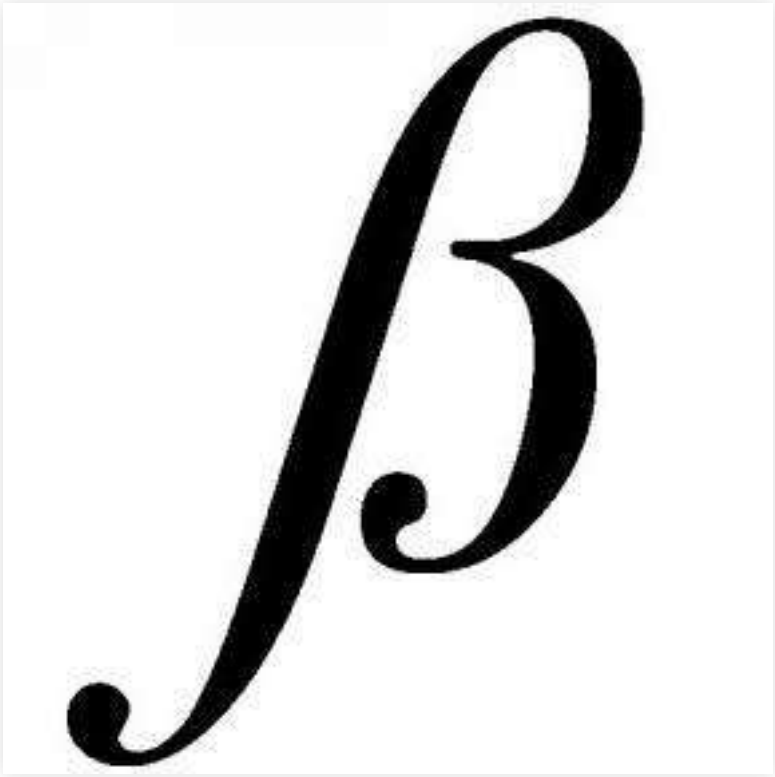
SYSTEMATIC RISK PRINCIPLE

There is a reward for bearing risk; There is not a reward for bearing risk unnecessarily. The expected return on a risky asset depends only on that asset's systematic risk since unsystematic risk can be diversified away.

MEASURING SYSTEMATIC RISK

- How do we measure systematic risk?
  - We use the beta coefficient
- What does beta tell us?
  - A beta of 1 implies the asset has the same systematic risk as the overall market
  - A beta < 1 implies the asset has less systematic risk than the overall market
  - A beta > 1 implies the asset has more systematic risk than the overall market

CURRENT BETA'S



TOTAL VS. SYSTEMATIC RISK

Consider the following information:

	Standard Deviation	Beta
Marathon Oil	20%	3.13
Exxon Mobil	30%	0.69

- Which security has more total risk? Exxon Mobil
- Which security has more systematic risk? Marathon Oil
- Which security should have the higher expected return? Marathon Oil

PORTFOLIO BETA

Consider the previous example with the following four securities

Security	Weight	Beta
DIS	.133	1.444
KO	.2	0.797
AAPl	.267	1.472
PG	.4	0.647

What is the portfolio beta?

PORTFOLIO BETA

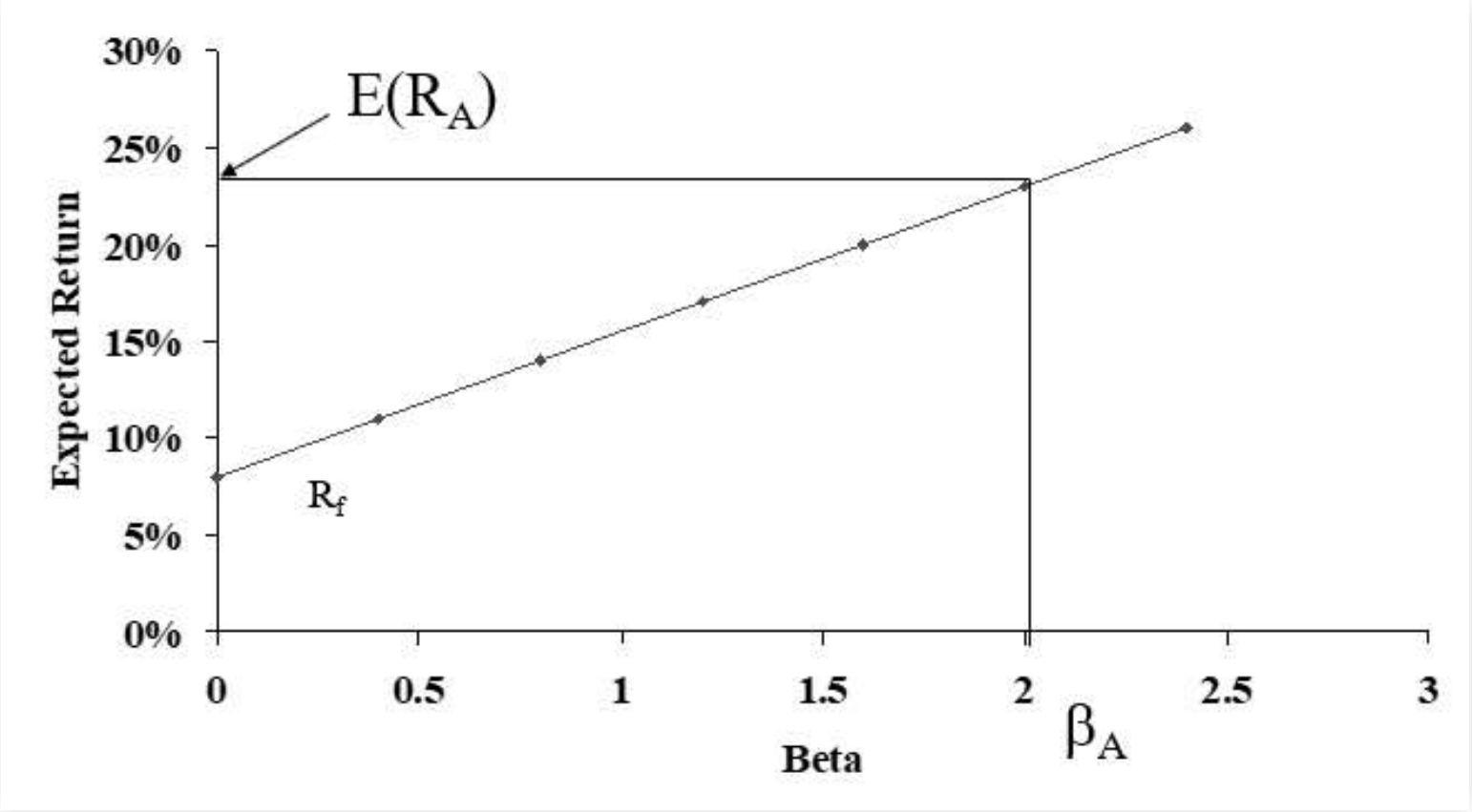
Consider the previous example with the following four securities

Security	Weight	Beta
DIS	.133	1.444
KO	.2	0.797
AAPl	.267	1.472
PG	.4	0.647

What is the portfolio beta?

$$.133(1.444) + .2(0.797) + .267(1.472) + .4(0.647) = 1.003$$

PORTFOLIO EXPECTED RETURNS AND BETAS



REWARD-TO-RISK RATIO

- The reward-to-risk ratio is the slope of the line illustrated in the previous example
  - $Slope = \frac{E(R_A) - R_f}{\beta_A - 0}$
  - From graph, $Slope = \frac{23-8}{2-0} = 7.5$
- What if an asset has a reward-to-risk ratio of 8 (implying that the asset plots above the line)?
- What if an asset has a reward-to-risk ratio of 7 (implying that the asset plots below the line)?

MARKET EQUILIBRIUM

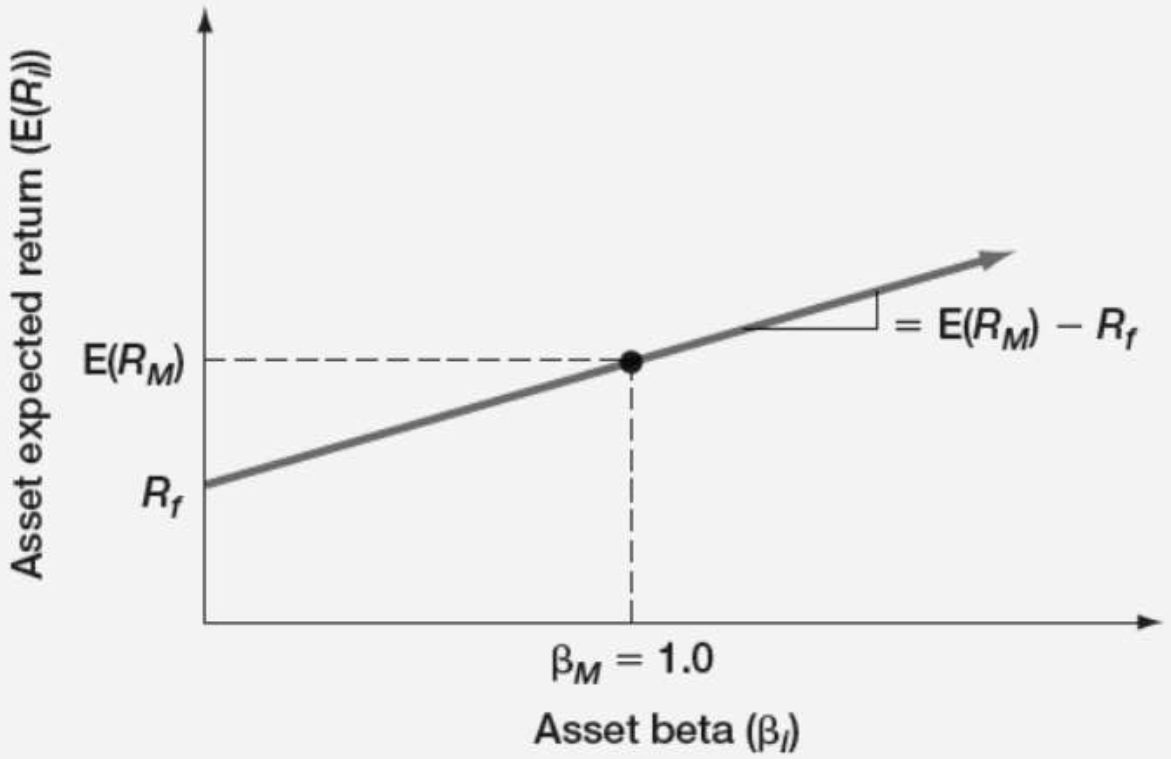
In equilibrium, all assets and portfolios must have the same reward-to-risk ratio, and they all must equal the reward-to-risk ratio for the market

$$\frac{E(R_A)-R_f}{\beta_A} = \frac{E(R_M)-R_f}{\beta_M}$$

SECURITY MARKET LINE

- The security market line (SML) is the representation of market equilibrium
- The slope of the SML is the reward-to-risk ratio: $\frac{E(R_M)-R_f}{\beta_M}$
- But since the beta for the market is always equal to one, the slope can be rewritten
- Slope $= E(R_M)-R_f =$ market risk premium

PUT IT ALL TOGETHER...



The slope of the security market line is equal to the market risk premium—that is, the reward for bearing an average amount of systematic risk. The equation describing the SML can be written:

$$E(R_i) = R_f + [E(R_M) - R_f] \times \beta_i$$

which is the capital asset pricing model (CAPM).

THE CAPITAL ASSET PRICING MODEL (CAPM)

The capital asset pricing model defines the relationship between risk and return

$$E(R_i) = R_f + \beta_i(E(R_M) - R_f)$$

- If we know an asset's systematic risk, we can use the CAPM to determine its expected return
- This is true whether we are talking about financial assets or physical assets

FACTORS AFFECTING EXPECTED RETURN

1. Pure time value of money: measured by the risk-free rate
2. Reward for bearing systematic risk: measured by the market risk premium
3. Amount of systematic risk: measured by beta

CAPM: EXAMPLE

Consider the betas for each of the assets given earlier. If the risk-free rate is 4.15% and the market risk premium is 8.5%, what is the expected return for each?

Asset	Beta	$E(R_i)$
DIS	1.444	4.15 + 1.444(8.5) = 16.42%
KO	0.797	4.15 + 0.797(8.5) = 10.92%
AAPL	1.472	4.15 + 1.472(8.5) = 16.66%
PG	0.647	4.15 + 0.647(8.5) = 9.65%

EXTRA PRACTICE

EXAMPLE 1

One year ago, Avril purchased 3,600 shares of Lavigne stock for $101,124. Today, she sold those shares for $26.60 a share. What is the total return on this investment if the dividend yield is 1.7 percent?

EXAMPLE 2

A stock has yielded returns of 6 percent, 11 percent, 14 percent, and -2 percent over the past 4 years, respectively. What is the standard deviation of these returns?

EXAMPLE 3

You purchased 1,300 shares of LKL stock 5 years ago and have earned annual returns of 7.1 percent, 11.2 percent, 3.6 percent, -4.7 percent and 11.8 percent. What is your arithmetic average return?What is the geometric return?

EXAMPLE 4

What is the expected return, variance, and standard deviation?

State	Probability	Go Nuts for Donuts Inc.
Boom	.25	.15
Normal	.5	.08
Slowdown	.15	.04
Recession	.10	-.03

EXAMPLE 5

Consider the following information on returns and probabilities:

State	Probability	Apple	Disney
Boom	.25	15%	10%
Normal	.6	10%	9%
Recession	.15	5%	10%

What are the expected return and standard deviation for a portfolio with an investment of $6,000 in Apple and $4,000 in Disney?

EXAMPLE 6

The risk free rate is 4%, and the required return on the market is 12%.

1. What is the required return on an asset with a beta of 1.5?
2. What is the reward/risk ratio?
3. What is the required return on a portfolio consisting of 40% of the asset above and the rest in an asset with an average amount of systematic risk?

KEY LEARNING OUTCOMES

- First Lesson: Average Returns
  - Historical returns
  - Risk Premium
- Second Lesson: Return Variability
  - Standard deviation
- Arithmetic vs Geometric return
- Capital market efficiency
  - Efficient market hypothesis

KEY LEARNING OUTCOMES

- Calculate:
  - Expected return, variance, and standard deviation
  - Do so for a portfolio of assets
- Understand diversification
  - Total risk, Systematic risk, Unsystematic risk
- Beta, Security Market Line and CAPM
  - Understand concept and derivation
  - Calculate Portfolio Beta
  - Use CAPM